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A Comment on Brane Bending and Ghosts in Theories with Infinite Extra Dimensions

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Abstract

Theories with infinite volume extra dimensions open exciting opportunities for particle physics. We argued recently that along with attractive features there are phenomenological difficulties in this class of models. In fact, there is no graviton zero-mode in this case and 4D gravity is obtained by means of continuum bulk modes. These modes have additional degrees of freedom which do not decouple at low energies and lead to inconsistent predictions for light bending and the precession of Mercury's perihelion. In a recent papers, [hep-th/0003020] and [hep-th/0003045] the authors made use of brane bending in order to cancel the unwanted physical polarization of gravitons. In this note we point out that this mechanism does not solve the problem since it uses a *ghost* which cancels the extra degrees of freedom. In order to have a consistent model the ghost should be eliminated. As soon as this is done, 4D gravity becomes unconventional and contradicts General Relativity. New mechanisms are needed to cure these models. We also comment on the possible decoupling of the ghost at large distances due to an apparent flat-5D nature of space-time and on the link between the presence of ghosts and the violation of positive-energy conditions.

Theories with infinite volume extra dimensions open exciting opportunities for particle physics. The following 5D warped metric may serve as a good example of this class of models:

$$ds^2 \ = \ A(y)\eta_{\mu\nu} \, dx^\mu dx^\nu \ - \ dy^2 \ , \tag{1}$$

where the warp factor $A(y)$ tends to a *nonzero* constant at $\pm\infty$. A brane setup which realizes this was recently proposed in Ref. [1]. It was argued in Refs. [2] and [3] that these models are very attractive since they could give new insights into bulk supersymmetry and the cosmological constant problem. Regretfully, as they stand right now, these theories face two serious challenges: to reproduce the correct four-dimensional Einstein limit without invoking ghost states [2], and to satisfy a weak energy positivity condition [3].

The aim of the present note is to respond to the criticism of [4] and [5] regarding the first issue.

Let us first recall the arguments of Ref. [2]. The work was based on the following assumptions:

I) The theory is self-consistent, in the sense that it has no *unconventional* or unphysical states, such as ghosts;

II) 5D gravity couples universally to the energy-momentum tensor $T_{\mu\nu}$.

We argue that in [4] and [5] the condition (I) is relaxed.

In models with infinite extra dimensions, differently form the Randall-Sundrum (RS) model [6], there is no localized 4D spin-2 or spin-0 zero-mode. The only relevant physical degrees of freedom are 4D massive spin-2 gravitons. As a result, 4D gravity is obtained by exchanging a metastable graviton [1, 7, 2]. This is equivalent to the exchange of a continuum of massive spin-2 bulk states. Each of the continuum states, from the 4D point of view, has strictly 5 physical degrees of freedom. They can be conveniently decomposed as: 2 from the 4D massless graviton, 2 from the "graviphoton" and 1 from a "graviscalar". Two of these, coming from the "graviphotons" are not relevant for matter localized on the brane. Graviscalars contribute to physical processes [8, 9]. These extra scalar degrees of freedom lead to deviations from the standard predictions of Einstein's theory [8, 9, 2] since the tensor structures of massive and massless graviton propagators are different:

$$\begin{aligned}
&\left(\frac{1}{2}(\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha})-\frac{1}{3}\eta^{\mu\nu}\eta^{\alpha\beta}+\mathcal{O}(p)\right) \qquad \text{massive ;}\\
&\left(\frac{1}{2}(\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha})-\frac{1}{2}\eta^{\mu\nu}\eta^{\alpha\beta}+\mathcal{O}(p)\right) \qquad \text{massless .}
\end{aligned} \tag{2}$$

Under assumptions (I) and (II), the 4D gravitational interactions are completely determined by the exchange of bulk gravitons. As we emphasized above, from the 4D point of view, these are just massive spin-2 states, with 5 degrees of freedom for each of them. The effective 4D gravity in [1] is obtained by summing up these states. Therefore, it is clear that the degrees of freedom do not match with those of 4D General Relativity and lead to unacceptable predictions [2]. In other words,

there is an additional scalar degree of freedom in the 4D world obtained in [1]. Note that our arguments are very general and are based only on the assumption of unbroken 4D general covariance.

The way to evade this result is to compensate the extra scalar with a ghost state. Clearly, if one introduces unconventional states, such as ghosts [4, 5], the results of [2] are modified, but then it is hard to make sense of the theory (see discussions below).

It was suggested in Ref. [4] that the unwanted polarizations are canceled if brane bending is taken into account. The question is how one can reconcile this claim with the 4D arguments presented above? The only way is by relaxing the assumption (I) of Ref. [2] and allowing for a ghost state in the theory.

To see that this is indeed the case in [4] let us recall that the brane bending studied in [10] and [11] is just a gauge choice which is needed to maintain the linearized approximation. A detailed formalism was developed in Refs. [10, 11], and was reiterated in Refs. [4] and [5] for the particular case at hand, so we won't be repeating it here. We just point out that the brane bending reveals a ghost field which is used in [4] to cancel the unwanted graviton polarizations. A most simple way to see this is as follows. Suppose that a brane with no matter is located at $y = 0$. After a matter source is introduced on the brane, its location is shifted to $y - \zeta(x)$, where $\zeta(x)$ is some response function determined by the source. Thus, matter couples to 4D fluctuations through the warp factor $A(y - \zeta(x))$. Expanding $A(y - \zeta(x))$ in powers of ζ one finds an additional coupling of $T^{\mu\nu}$ to $\zeta\partial A$. This is the coupling which effectively introduces a ghost. Indeed, let us introduces a source with energy-momentum tensor

$$T_{\mu\nu} \ \equiv \ S_{\mu\nu} \ \delta(\bar{y}) \ . \tag{3}$$

When the brane is bent by the matter source, one may choose new coordinates $\bar{x}$, $\bar{y}$ using a gauge transformations (see for details [10, 11, 4, 5]). The induced metric on the brane takes the following form in these coordinates:

$$\bar{h}_{\mu\nu}(x,0) \propto \int d^4z \left(D_5(x,0;z,0)(S_{\mu\nu}(z) - \frac{1}{3}\eta_{\mu\nu}S^\alpha_\alpha(z)) - H\eta_{\mu\nu}D_4(x,z)S^\alpha_\alpha(z) \right) \ , \tag{4}$$

where D_5 denotes the scalar part of the 5D graviton propagator, D_4 denotes that of a four-dimensional scalar and H is some positive constant proportional to the square root of the bulk cosmological constant. The last term in this expressions is equivalent to a contribution of a scalar ghost field. We would like to point out here that brane bending term does not cause any problem in the RS scenario. Moreover, it is needed for self-consistency of the RS model. Recall that in the RS framework there is a massless graviton zero-mode with 2 physical degrees of freedom, in addition there is an unphysical "graviscalar" which is gauge dependent, plus there are massive spin-2 gravitons. The ghost in the RS framework is explicitly canceled by an *unphysical* "graviscalar". Therefore, one is left with the 2 physical polarizations of the 4D massless graviton zero-mode. One might think of this as canceling

a longitudinal photon by A_0 "ghost" in the Gupta-Bleuler quantization of QED. This cancellation of unphysical states does not take place if there is no localized zero-mode. Which is precisely what happens in [1]. As we discussed above, states which mediate 4D gravity in this case are just massive spin-2 states. They have 5 degrees of freedom which are *all physical*. Two degrees of freedom corresponding to "graviphotons" decouple at low energies, as they couple derivatively to a conserved energy-momentum tensor. However, the third physical scalar does not decouple. The aim of the ghost present in [4] is to compensate for this scalar. Thus, one is left with the theory which has a manifest *ghost* in the physical spectrum. This ghost was used in [5] to remove the problem of extra degrees of freedom from the 4D theory to large distances, where gravity, in this case, becomes scalar antigravity due to the ghost.

However, the presence of a ghost indicates sickness of a theory at *any* scales. In particular, the ghost energy is unbounded from below. Any theory which looks remotely like gravity is then completely unstable when coupled to such a state. This instability is most probably due to the fact that the background in [1] violates positive-energy conditions [3].

Since there are no known ways to remedy theories with physical ghosts, we are inclined to take a conservative point of view and require that ghost contributions should be canceled for a sensible model. In this case the model can be made free of ghosts, however, the gravity in 4D becomes a tensor-scalar gravity and one goes back to the problems pointed out in Ref. [2].

One may wonder whether the ghost can persist at large distances. This is a bit confusing, since it naively seems that the model of Ref. [1] should become flat-five-dimensional at large distances in which case the second term in (4) is clearly absent. However, the theory at hand is never a flat-five-dimensional one. Rather it is a flat-five-dimensional model with a peculiar brane. This brane is a combination of a positive and negative tension slices and from large distances looks as a zero tension object. However, regardless of the fact that this is a zero-tension object, it brakes *maximally* translation invariance in extra dimensions. As a result, there is no continuous limit in which the theory is flat-five-dimensional.

Another possibility is to make the ghost metastable, and decay at large distances. Still, even in this case, it should admit a Källen-Lehman representation in terms of massive ghost states. Again, metastability does not suffice to "exorcise" the ghost. Indeed, this may at most cure problems in single-ghost exchange amplitudes, but not in amplitudes involving two or more ghosts. The instability associated with the fact that the ghost energy is unbounded-below is just an example.

The ghost formulation of the problems raised in [2] makes us think that they might be related to the lack of energy-positivity in this scenario [3]. Probably, any solution of the ghost problem must also cure the energy-positivity problem. In any event, this framework deserves further investigation and perhaps there are some unconventional solutions to the problems discussed above.

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